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Time Scales in Quantum Mechanics by a Scattering Map

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In quantum mechanics the problem of decoherence for an isolated, finite system is linked to a coarse-grained description of its dynamics.

1. INTRODUCTION

As pointed out long ago by Ludwig, the typical problems about the foundations of quantum mechanics (QM), basically arising from nonseparability, such as the EPR paradox and macroscopic superpositions in measuring processes, are avoided if one shifts the basic elements of reality with which the theory is dealing from microphysical components of matter to the macroscopic setup of any experiment; then microsystems are derived objects carrying correlations and interactions between sources and detectors. The microphysical structure of matter operatively implies the existence of this utmost simple interaction channel between systems, and QM is just its beautifully simple theory that one can derive from axioms superposed on an objective description of macrosystems (Ludwig, 1983), arriving in this way at the modern formulation in terms of POV measures and instruments (Kraus, 1983; Holevo, 1982; Davies, 1976). So the real challenge of QM is the objective description of macrosystems, where we are giving this name to any part of the world that is separated from the environment and prepared in such a way that some objective, and in this sense classical, description of it can be given. Adopting this viewpoint has very relevant consequences for the mathematical setting of the theory. Looking at any realistic example of a quantum description of a macrosystem, there can be no doubt that its preparation has to be described

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by a mixture and not by a pure state. Preparations of the system are elements of the set $\mathcal{K}(\mathcal{H})$ of statistical operators on the Hilbert space $\mathcal{H}$, which is naturally located in the Banach space $\mathcal{T}(\mathcal{H})$ of trace-class operators [it is the base of the positive cone, and the whole space $\mathcal{T}(\mathcal{H})$ is "positively generated" by it (Davies, 1976)]. Then the most natural way of representing the transformation of a preparation, like the very fundamental one arising by the time evolution of an isolated system, is by means of a mapping $\mathcal{M}$ on $\mathcal{T}(\mathcal{H})$ transforming $\mathcal{K}$ into $\mathcal{K}$. However, such a mapping is not in general an isometry; this happens if and only if it has an inverse and it is then implemented by a unitary mapping on $\mathcal{H}$ ($\mathcal{M} \cdot = \hat{U} \cdot \hat{U}^{\dagger}$), otherwise one meets a truly more general framework for the dynamics, in which irreversibility appears as the typical new feature. On the contrary, if one takes as the starting point the QM of microsystems, the use of pure states is usually a very appropriate idealization, justified by the high level of experimental control by which a few-particle system can be prepared and strongly supported by the outstanding role of unitary representations of symmetries in particle physics: in this context time evolution is described in the most natural way by the Schrödinger equation. Tackling the question of macrosystems, one comes to statistical operators invoking incomplete information about initial state, or decoherence by the environment, or some mathematical extrapolation to an infinite system. The last point of view gives nice results, but only in the particular case of systems at equilibrium. So the usual scenario for the extraordinary performances of QM is not very satisfactory: no direct objectivity can be attributed to particles, and macrosystems (not at equilibrium), by which in Bohr's philosophy such objectivity can be recovered, can be described only in a thermodynamic limit, which is hardly compatible with nonequilibrium situations.

2. DEALING WITH FINITE MACROSYSTEMS

The very concept of isolated macrosystem is slippery: the macrosystem must be separated inside a spatial region ω by a suitable preparation procedure covering a finite time interval $[T, t_0]$, the "preparation time." We will not take the limit $T \to -\infty$, since in our opinion one should avoid shifting this problem to a cosmological level. Considering a finite preparation time means that some memory loss is operatively necessary, the price of some coarse graining of the dynamical description must be paid: to do this we associate in a systematic way a suitable time scale to the preparation procedure. The relevant role of the preparation procedure means a breaking of basic space-time symmetry by suitable boundary conditions which introduce the peculiarities of the system, hiding the more universal behavior of local or short-range interactions. The field-theoretic approach, which is anyway mandatory in the

relativistic case, is best suited to express the interplay of local universality and peculiar boundary conditions. In this discussion of macrosystems let us take, in the nonrelativistic limit, a very schematic model, built by one type of molecules confined inside a region ω and interacting by a two-body potential $V(|\mathbf{x}-\mathbf{y}|)$; this system is described by a quantum Schrödinger field (QSF) $\hat{\psi}(\mathbf{x})$, to which the following local Hamiltonian density is associated:

$$\hat{e}(\mathbf{x}) = \frac{\hbar^2}{2m}\nabla\hat{\psi}^\dagger(\mathbf{x})\cdot\nabla\Delta\hat{\psi}(\mathbf{x}) + \frac{1}{2}\int_\omega d^3\mathbf{y}\ \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{y})V(|\mathbf{x}-\mathbf{y}|)\hat{\psi}(\mathbf{y})\hat{\psi}(\mathbf{x}) \qquad (2.1)$$

$$[\hat{\psi}(\mathbf{x}), \hat{\psi}^\dagger(\mathbf{x}')]_\pm = \delta(\mathbf{x}-\mathbf{x}')$$

Let us consider a complete orthonormal set of eigenfunctions $u_f \in L^2(\omega)$,

$$-\frac{\hbar^2}{2m}\Delta_2 u_f(\mathbf{x}) = W_f u_f(\mathbf{x}) \qquad \mathbf{x}\in\omega, \qquad u_f(\mathbf{x}) = 0 \qquad \mathbf{x}\in\partial\omega \quad (2.2)$$

corresponding to the numerable set of eigenvalues W_f, and the confined QSF $\hat{\psi}_C(\mathbf{x}) = \Sigma_f u_f(\mathbf{x})\hat{a}_f$. It will replace $\hat{\psi}(\mathbf{x})$ in (2.1) and we shall leave out the subscript C. The Hamiltonian $\hat{H}$ and mass operator $\hat{M}$ will be taken respectively as

$$\hat{H} = \int_\omega d^3\mathbf{x}\ \hat{e}(\mathbf{x}), \qquad \hat{M} = \int_\omega d^3\mathbf{x}\ \hat{\rho}_m(\mathbf{x}), \qquad \hat{\rho}_m(\mathbf{x}) = m\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x}) \qquad (2.3)$$

Obviously it may be uncomfortable to deal with the functions $u_f(\mathbf{x})$ and to perform discrete sums, even if, but only at a final stage, one can do approximations like

$$\sum_f h(W_f)\ u_f(\mathbf{x}) = \int d\mu(\mathbf{p})\ h\left(\frac{\mathbf{p}^2}{2m}\right)e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}$$

The time scale is related to the choice of the relevant fields in terms of which $\hat{e}(\mathbf{x})$ is given. The picture founded on a "mass charged" field associated to molecules holds if the physics of the system essentially depends on elastic scattering of neutral molecules, the whole underlying electromagnetic structure being hidden: the intermolecular (e.g., Lennard-Jones) potential $V(r)$ is a simple effective representation of the molecular field self-interaction. A much deeper description of dynamics is possible in terms of "electrically charged" fields (electron and nuclei) based on QED, but also in this case effective rough elements will enter in the Hamiltonian density, e.g., the

electromagnetic form factors of nuclei. Unfortunately, no systematic attempt to base macrophysics on QED has been developed. One can expect that the relevance of time scales in macrophysics, the increasingly deeper descriptions lowering the time scale, even if at any stage the separation procedure requires a persistence of some coarse graining of the dynamical description, indicates a link with the ultraviolet renormalization problem in field theory: such a link appears clearer if quantum field theory is seen as the basic theory of macroystems, rather than of particles. Let us now indicate briefly how a piece of macrophysics can be built based on QSF: hydrodynamics, or with a slight generalization, kinetic description of a massive neutral continuum. First of all a classical velocity field is associated to the continuum and the following basic densities of conserved quantum observables are considered:

$$\hat{e}^{(0)}(\mathbf{x}) = \frac{1}{2m}(i\hbar\nabla - m\mathbf{v}(\mathbf{x}, t))\hat{\psi}^{\dagger}(\mathbf{x}) \cdot (-i\hbar\nabla - m\mathbf{v}(\mathbf{x}, t))\hat{\psi}(\mathbf{x}) + \frac{1}{2}\int_{\omega} d^3\mathbf{y}\ \hat{\psi}^{\dagger}(\mathbf{x})\hat{\psi}^{\dagger}(\mathbf{y})V(|\mathbf{x} - \mathbf{y}|)\hat{\psi}(\mathbf{y})\hat{\psi}(\mathbf{x})$$

$$\hat{\rho}_m^{(0)}(\mathbf{x}) = \hat{\rho}_m(\mathbf{x}) \tag{2.4}$$

The field $\hat{e}^{(0)}(\mathbf{x})$ represents the energy density in the reference frame in which the continuum is locally at rest. In the kinetic description the mass density $\hat{\rho}_m(\mathbf{x})$ should be replaced by the more detailed phase-space distribution observable

$$\hat{f}(\mathbf{x}, \mathbf{p}) = m\sum_{hk} \hat{a}_h^{\dagger}\langle u_h|\hat{\mathsf{F}}^{(1)}(\mathbf{x}, \mathbf{p})|u_k\rangle\hat{a}_k, \qquad \hat{M} = \int_{\omega} d^3\mathbf{x}\int_{R^3} d^3\mathbf{p}\ \hat{f}(\mathbf{x}, \mathbf{p})$$

constructed on the second-quantized form of the operator density $\hat{\mathsf{F}}^{(1)}(\mathbf{x}, \mathbf{p})$ on $\omega \times R^3$ yielding a joint position–momentum observable. In correspondence to the velocity field $\mathbf{v}(\mathbf{x}, t)$ and to functions $e^{(0)}(\mathbf{x}, t)$ and $\rho_m^{(0)}(\mathbf{x}, t)$ associated at time t to the operator fields $\hat{e}^{(0)}(\mathbf{x}, t)$ and $\hat{\rho}_m^{(0)}(\mathbf{x}, t)$, one considers the subset of $\mathcal{H}$ such that

$$e^{(0)}(\mathbf{x}, t) = \mathrm{Tr}(\hat{e}^{(0)}(\mathbf{x})\hat{w}), \qquad \rho_m(\mathbf{x}, t) = \mathrm{Tr}(\hat{\rho}_m(\mathbf{x})\hat{w}), \qquad 0 = \mathrm{Tr}(\hat{\mathbf{p}}^{(0)}(\mathbf{x})\hat{w}) \tag{2.5}$$

with $\hat{\mathbf{p}}^{(0)}(\mathbf{x})$ the momentum density observable in the reference frame locally at rest

$$\hat{\mathbf{p}}^{(0)}(\mathbf{x}) = \frac{1}{2}\{[(i\hbar\nabla - m\mathbf{v}(\mathbf{x}, t))\hat{\psi}^{\dagger}(\mathbf{x})]\hat{\psi}(\mathbf{x}) - \hat{\psi}^{\dagger}(\mathbf{x})(i\hbar\nabla + m\mathbf{v}(\mathbf{x}, t))\hat{\psi}(\mathbf{x})\}$$

Then one looks for an element of this subset such that its von Neumann entropy $S = -k\mathrm{Tr}\ (\hat{w}\ \log\ \hat{w})$ is maximal. This means a statistical operator giving the assigned classical state with highest mixture: i.e., assigning the classical state has an unmixing role, but no other unmixing process is supposed. This leads to a generalized Gibbs state (Robin, 1990):

$$\hat{w}_G(t) \equiv \hat{w}[\beta(t),\ \mu(t),\ \mathbf{v}(t)]$$

$$= \frac{\exp\left\{-\int_\omega d^3\mathbf{x}\ \beta(\mathbf{x},\ t)[\hat{e}^{(0)}(\mathbf{x}) - \mu(\mathbf{x},\ t)\hat{\rho}_m(\mathbf{x})]\right\}}{\mathrm{Tr}\exp\left\{-\int_\omega d^3\mathbf{x}\ \beta(\mathbf{x},\ t)[\hat{e}^{(0)}(\mathbf{x}) - \mu(\mathbf{x},\ t)\hat{\rho}_m(\mathbf{x})]\right\}} \tag{2.6}$$

In the kinetic case $\hat{\rho}_m$ is replaced by $\hat{f}$ and μ is a function $\mu\ (\mathbf{x},\ \mathbf{p})$. The parameters $\beta(t)$, $\mu(t)$, $\mathbf{v}(t)$ are determined by (2.5) and will be considered as objective state variables of the macrosystem; $S = -k\mathrm{Tr}\hat{w}_G(t)\ \log\ \hat{w}_G(t)$ is the entropy of the system.

Now the problem arises to make a suitable choice for the representative of the state at some initial time t_0. According to "information thermodynamics," one takes the generalized Gibbs state determined by the given expectation values at time t_0, which is the most unbiased choice. This approach is certainly satisfying if memory effects are absent or completely negligible and if no other information about the system, apart from these expectations, is available, that is to say: the preparation procedure may be idealized by the instantaneous measurement of the relevant variables. More general situations, for example, memory effects connected to a microphysical correlation time, demand a preparation procedure covering at least the correlation time, thus leading to memory terms in the representative of the state. The dynamical evolution law must then be fine enough to take such effects into account. To circumvent these difficulties Zubarev, in his definition of the "non-equilibrium statistical operator" (Zubarev, 1974), takes the limit $t_0 \to -\infty$, thus removing any possible previous memory. This is obtained at the price of introducing a weighting factor $e^{\varepsilon t}$ that has to be eliminated after the thermodynamic limit has been taken, thus resorting once more to an infinite limit. Anyway a suitable memory loss mechanism must be still assumed, typically the decay time of correlation functions. Our aim is to extract from the dynamics this memory loss mechanism, related to a time scale and described inside the more general framework that we have indicated.

3. TIME SCALE AND SCATTERING MAP

Let us apply the model described in Section 2 to a dilute gas, assuming that it has been prepared so that the relevant variables $\langle\hat{e}(\mathbf{x})\rangle$, $\langle\hat{\rho}_m(\mathbf{x})\rangle$, and

$\mathbf{v}(\mathbf{x}, t)\,\langle\hat{\rho}_m(\mathbf{x})\rangle = \langle\hat{\mathbf{p}}(\mathbf{x})\rangle$ are smooth enough to provide a macroscopic variation time much larger than the microscopic collision time τ_0; then, taking into account the field-theoretic structure of the relevant observables, one has to study expressions of the form $\mathcal{U}'\,(\hat{a}_h^\dagger \hat{a}_k)$, $\mathcal{U}'\,(\hat{a}_{h_1}^\dagger \hat{a}_{h_2}^\dagger \hat{a}_{k_2} \hat{a}_{k_1})$, with $\mathcal{U}'$ being the time evolution mapping in Heisenberg picture on $\mathcal{B}(\mathcal{H})$ ($\mathcal{U}' \cdot = e^{+(i/\hbar)\hat{H}t} \cdot e^{-(i/\hbar)\hat{H}t}$), and to look for an asymptotic representation for $t >> \tau_0$. Our procedure essentially consists in transferring to $\mathcal{B}(\mathcal{H})$ standard methods of scattering theory related to $\mathcal{H}$, so the following formulas need no other comment:

$$\mathcal{H}_0' = \frac{i}{\hbar}\,[\hat{H}_0, \cdot], \qquad \hat{H}_0 = \sum_f W_f \hat{a}_f^\dagger \hat{a}_f$$

$$\begin{aligned}\mathcal{U}'(t)(\hat{a}_h^\dagger \hat{a}_k) &= (\mathcal{U}'(t)\hat{a}_h^\dagger)(\mathcal{U}'(t)\hat{a}_k) \\ &= \int_{-i\infty+\eta}^{+i\infty+\eta} \frac{dz_1}{2\pi i}\, e^{z_1 t}\left(\frac{1}{z_1 - \mathcal{H}'}\,\hat{a}_h^\dagger\right) \int_{-i\infty+\eta}^{+i\infty+\eta} \frac{dz_2}{2\pi i}\, e^{z_2 t}\left(\frac{1}{z_2 - \mathcal{H}'}\,\hat{a}_k\right) \qquad (3.1)\end{aligned}$$

$$\frac{1}{z - \mathcal{H}'} = \frac{1}{z - \mathcal{H}_0'} + \frac{1}{z - \mathcal{H}_0'}\,\mathcal{T}(z)\,\frac{1}{z - \mathcal{H}_0'}$$

$$\mathcal{T}(z) \equiv \mathcal{V}' + \mathcal{V}'\,\frac{1}{z - \mathcal{H}'}\,\mathcal{V}'$$

$\mathcal{T}(z)$, reminiscent of the T-matrix, plays a central role in this treatment: it will be called "scattering map." Existence of τ_0 means suitable smoothness properties of $\mathcal{T}(z)$, so that essentially only the poles of $(z - \mathcal{H}_0')^{-1}$ contribute to the calculation in (3.1), leading to the representation

$$\mathcal{U}'(t)(\hat{a}_h^\dagger \hat{a}_k) = \hat{a}_h^\dagger \hat{a}_k + t\mathcal{L}'(\hat{a}_h^\dagger \hat{a}_k), \qquad \tau_0 << t << \frac{\hbar}{|E_h - E_k|} \qquad (3.2)$$

Analogous formulas should be written with $\hat{a}_{h_1}^\dagger \hat{a}_{h_2}^\dagger \hat{a}_{k_2} \hat{a}_{k_1}$ in place of $\hat{a}_h^\dagger \hat{a}_k$; for brevity we skip the derivation of (3.2), stressing only the structural features. $\mathcal{L}'$ is a linear mapping in $\mathcal{B}(\mathcal{H})$ initially defined on the family of linearly independent elements $\hat{a}_h^\dagger \hat{a}_k$, $\hat{a}_{h_1}^\dagger \hat{a}_{h_2}^\dagger \hat{a}_{k_2} \hat{a}_{k_1}$. Our approach, related to relevant field variables in Heisenberg picture, differs strongly from master equation theory or investigations of subdynamics (e.g., Prigogine's approach) aiming at a subdynamics for the statistical operator. The definition of $\mathcal{L}'\,(\hat{a}_h^\dagger \hat{a}_k)$, the operator in the Fock-space of QSF, is at first sight very simple:

$$\begin{aligned}\mathcal{L}'(\hat{a}_h^\dagger \hat{a}_k) = \frac{i}{\hbar}\,[\hat{H}_{\text{eff}}, \hat{a}_h^\dagger \hat{a}_k] - \frac{1}{\hbar}\,([\hat{\Gamma}^{(2)}, \hat{a}_h^\dagger]\hat{a}_k - \hat{a}_h^\dagger[\hat{\Gamma}^{(2)}, \hat{a}_k]) \\ + \frac{1}{\hbar}\sum_\lambda \hat{R}_{h\lambda}^{(2)\dagger} \hat{R}_{k\lambda}^{(2)}\end{aligned}$$

$$\hat{H}_{\text{eff}} = \sum_f W_f \hat{a}_f^\dagger \hat{a}_f + \frac{1}{2} \sum_{\substack{l_1 l_2 \\ f_1 f_2}} \hat{a}_{l1}^\dagger \hat{a}_{l2}^\dagger V^{\text{eff}}_{l_1 l_2 f_2 f_1} \hat{a}_{f_2} \hat{a}_{f_1}$$

$$\hat{\Gamma}^{(2)} = \frac{1}{4} \sum_{h\lambda} \hat{R}^{(2)\dagger}_{h\lambda} \hat{R}^{(2)}_{h\lambda}$$

$$\hat{R}^{(2)}_{k\lambda} = \sum_{f_1 f_2} R_{k\lambda f_2 f_1} \hat{a}_{f_2} \hat{a}_{f_1} \tag{3.3}$$

The coefficients $V^{\text{eff}}_{l_1 l_2 f_2 f_1}$ and $R_{k\lambda f_2 f_1}$ are directly related to the two-particle T-matrix for scattering produced by the potential $V(r)$ appearing in (2.1). This simple structure comes from a "one-interacting-mode approximation," appropriate for a not too dense system, by which only two-particle collisions are considered. However, the definition of $\mathscr{L}'$ is trickier since by quantum nonseparability, Pauli principle corrections must arise. In fact the coefficients $V^{\text{eff}}_{l_1 l_2 f_2 f_1}$ and $R_{k\lambda f_2 f_1}$ are not c-numbers, but are operator valued in the Fock-space of QSF, diagonal in the basis created by $\hat{a}_f^\dagger$; to transform an element $| \ldots n_f \ldots \rangle$ of this basis by the operator $\mathscr{L}'$ $(\hat{a}_h^\dagger \hat{a}_k)$ one applies to it the r.h.s. of (3.3), where the coefficients $V^{\text{eff}}_{l_1 l_2 f_2 f_1}$ and $R_{k\lambda f_2 f_1}$ are functionals of the configuration $\{n_f\}$. $\mathscr{L}'$ as given by (3.3) generates a positive dynamics, i.e., $(\mathscr{I} + \tau\mathscr{L}')$ is positive at first order in τ, for $\tau > 0$. Actually one has a stronger property:

$$\sum_{hk} \langle \psi_h | [(\mathscr{I} + \tau\mathscr{L}')(\hat{a}_h^\dagger \hat{a}_k)] \psi_k \rangle > 0, \qquad \forall \{\psi_k\}, \quad \psi_k \in \mathscr{H} \tag{3.4}$$

Due to the fact that (3.4) holds for $\tau > 0$ only, irreversibility is introduced. Property (3.4) looks like a straightforward adjustment to the Fock-space structure of the well-known complete positivity notion for a mapping $\mathscr{M}$ on $\mathscr{B}(\mathscr{H})$:

$$\sum_{hk} \langle \psi_h | \mathscr{M}(\hat{A}_h^\dagger \hat{A}_k) \psi_k \rangle > 0 \qquad \forall \{\psi_k\}, \{\hat{A}_k\},$$

$$\psi_k \in \mathscr{H}, \quad \hat{A}_k \in \mathscr{B}(\mathscr{H})$$

The link between $\hat{\Gamma}^{(2)}$ and $R^{(2)}_{k\lambda}$ implies mass conservation, $\mathscr{L}'\hat{M} = 0$. Now the following assumption becomes very natural: the generalized Gibbs states related to the relevant observables $\hat{e}(\mathbf{x})$, $\hat{\rho}_m(\mathbf{x})$ can also be used to obtain the expectations of the "coarse-grained" time derivatives of these variables, i.e., $\hat{w}$ and $\mathscr{L}'$ are respectively state and evolution map tuned to the time scale $t >> \tau_0$. Then by (2.5) one has the evolution equation for the generalized Gibbs states:

$$\frac{d}{dt} \operatorname{Tr}(\hat{A}\hat{w}[\beta(t), \mu(t), \mathbf{v}(t)]) = \operatorname{Tr}((\mathscr{L}'\hat{A})\hat{w}[\beta(t), \mu(t), \mathbf{v}(t)]) \tag{3.5}$$

where $\hat{A} = \hat{e}^{(0)}(\mathbf{x})$, $\hat{\rho}_m^{(0)}(\mathbf{x})$, $\hat{\mathbf{p}}^{(0)}(\mathbf{x})$, thus providing a set of closed evolution equations for the objective state parameters $\beta(t)$, $\mu(t)$, $\mathbf{v}(t)$. Choosing $\hat{A} = \hat{a}_h^\dagger \hat{a}_k$, by inspection of the r.h.s. of (3.3) one can immediately recognize the relationship with Boltzmann equation: the last two terms of (3.3) have the typical form of a collision operator, the $\hat{\Gamma}^{(2)}$, $\hat{R}_{h\lambda}^{(2)\dagger} \cdot \hat{R}_{k\lambda}^{(2)}$ contributions being respectively the loss and the gain part. This description avoids any factorization of many-particle distribution functions. The dynamics on the coarse-grained time scale $t >> \tau_0$ loses any memory of previous states and is described by the irreversible map $\mathscr{L}'$. If the approximation leading to $\mathscr{L}'$ does not work, one expects that memory effects can appear and that the starting point could be shifted from (2.1) to the QED Hamiltonian. To conclude these considerations about an isolated macrosystem described by a statistical operator $\hat{\rho}_{\mathrm{M}}(t)$ and a Hamiltonian $\hat{H}_{\mathrm{M}}$, let us show how the simplest breaking of the isolation of this system leads to the concept of a microsystem. Consider a Hamiltonian $\hat{H}$ and a statistical operator $\hat{\rho}(t)$ of the form

$$\hat{H} = \hat{H}_0 + \hat{H}_M + \hat{V}, \qquad \hat{H}_0 = \sum_p E_p \hat{b}_p^\dagger \hat{b}_p, \qquad [\hat{b}_p, \hat{b}_p^\dagger]_\mp = \delta_{pq} \tag{3.6}$$

$$\hat{\rho}(t) = \sum_{qp} \hat{b}_q^\dagger \hat{\rho}_{\mathrm{M}}(t) \hat{b}_p \hat{\rho}_{qp}(t), \qquad \hat{b}_p \hat{\rho}_{\mathrm{M}} = 0 \tag{3.7}$$

Due to the condition $\hat{b}_{\mathrm{p}} \hat{\rho}_M = 0$, the QSF $\hat{\phi}(\mathbf{x}) = \Sigma_q \tilde{u}_q(\mathbf{x}) \hat{b}_q$ is either related to a different particle or to other modes $\tilde{u}_g$ (e.g., confinement in $\tilde{\omega} \supset \omega$) than those involved in (2.2). Indicating with $\hat{Q} = \Sigma_q \hat{b}_q^\dagger \hat{b}_q$ the related charge, one has $\hat{Q}\hat{\rho}_{\mathrm{M}} = 0$, $\hat{Q}\hat{\rho} = \hat{\rho}$: this indicates the "elementary" nature of the changes of $\hat{\rho}$. Under suitable conditions, the new relevant variables $\hat{A} = \Sigma_{h,k} \hat{b}_h^\dagger A_{hk} \hat{b}_k$ can be treated by the same procedure we have indicated before. Using the reduction formula

$$\mathrm{Tr}_{\mathscr{H}}(\hat{A}\hat{\rho}(t)) = \mathrm{Tr}_{\mathscr{H}^{(1)}}(\hat{\mathsf{A}}^{(1)} \hat{\rho}^{(1)}(t))$$

$$\hat{\mathsf{A}}^{(1)} = \sum_{hk} |u_h\rangle A_{hk} \langle u_k|, \qquad \tilde{\rho}^{(1)} = \sum_{qp} |u_q\rangle \hat{\rho}_{qp} \langle u_p|$$

$\mathscr{H}^{(1)}$ being the Hilbert space spanned by $\tilde{u}_q$, the reduced dynamics can be interpreted as a microsystem described in the Hilbert space $\mathscr{H}^{(1)}$, with observables $\hat{\mathsf{A}}^{(1)}$ and preparation $\hat{\rho}^{(1)}$. For $\hat{\rho}^{(1)}$ a master equation has been found (Lanz and Vacchini, 1997a,b) describing both the optical behavior associated to the analogous of $\hat{H}_{\mathrm{eff}}$ in (3.3) and a incoherent part related to the other part of $\mathscr{L}'$. The role of the first part is enhanced in the typical setup of particle interferometry and in this way one comes back to the one-particle Schrödinger equation; the second part describes Brownian motion and thermalization of the particle inside matter. We recall that an objective reinterpretation of the dynamics of the new variables due to the non-Hamiltonian evolution is

possible in terms of a statistical description of trajectories of the nonisolated particle (Lanz and Melsheimer, 1993); however, a systematic extension of this objectifying procedure to the relevant macroscopic variables considered in Section 1, for which in the present treatment only the expectation values have been considered, is an open question. The statistical operator (3.7) describes a microsystem + a macrosystem without a reaction of the microsystem on the macrosystem, so this is not yet enough to treat in the context of the theory of macrosystems the typical setting: source–detector.

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